SCORE: _____ / 30 POINTS

1. NO CALCULATORS ALLOWED
2. UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS
3. SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

MULTIPLE CHOICE. Consider the DEs SCORE: 3 / 3 PTS

[1] $(5r+1)\frac{dr}{d\theta} = 2\theta + 1$ [2] $x''y - y^2 = 2x$ [3] $(5w+1)du + (\ln w - 3u)dw = 0$

$\frac{du}{dw}$

(where w is the independent variable)

Which of the DE above are linear ? Circle the correct answer below.

(a) none are linear (b) only [1] is linear (c) only [2] is linear (d) only [3] is linear

(e) only [1] & [2] are linear (f) only [1] & [3] are linear 3 (g) only [2] & [3] are linear (h) all are linear

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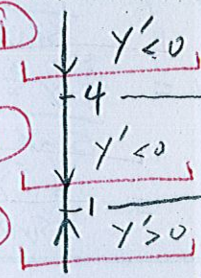
Consider the autonomous DE $y' = (y-4)^2(1-y)$. SCORE: 7 / 7 PTS

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.

$y' = 0$ $(y-4)^2(1-y) = 0$

$y-4=0$ $1-y=0$

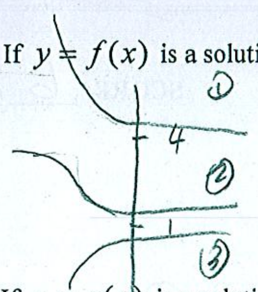
$y_1 = 4$ $y_2 = 1$ 1



semistable at $y=4$ 1

stable at $y=1$ 1

[b] If $y = f(x)$ is a solution of the DE such that $f(2) = 5$, what is $\lim_{x\to\infty} f(x)$? HINT: Sketch a possible graph of $y = f(x)$.



$f(2) = 5 > 4$

Which means it's in ①

that is $\lim_{x\to\infty} f(x) = 4$ 1

[c] If $y = g(x)$ is a solution of the DE such that $g(5) = -3$, what is $\lim_{x\to\infty} g(x)$?

$g(5) = -3 < 1$

the graph is in ③

$\lim_{x\to\infty} g(x) = 1$ 1

Write a differential equation for the velocity $v(t)$ of a falling object if the air resistance is proportional to the square of the velocity. Assume that $v(t) > 0$ corresponds to the object moving upward, $v(t) < 0$ corresponds to the object moving downward. (NOTE: This is NOT the same problem as in the homework.) SCORE: _____ / 3 PTS

$ma = -mg + kv^2_{\text{down}}$ ↑+

$\frac{dv}{dt} = \begin{cases} -g + \frac{kv^2}{m}, & v<0 \\ -g - \frac{kv^2}{m}, & v>0 \end{cases}$

1 $m\frac{dv}{dt} = -mg + kv^2$ 1

$\frac{dv}{dt} = -g + \frac{kv^2}{m}$, upward $k<0$, downward $k>0$.

FILL IN THE BLANKS. SCORE: 3 / 3 PTS

[a] The order of the DE $y^{10} - y^{(7)}y^4 = (x^5 + y''')^6$ is 7. 1

[b] If $y = \sqrt{x+9}$ is a solution of the DE $y'' = f(x, y, y')$, the largest possible interval of definition is $(-9, +\infty)$. 2

$\frac{1}{2\sqrt{x+9}}$

Consider the IVP $y' = 5x - 10y, \quad y(1) = -2$. SCORE: ____ / 5 PTS

Use Euler's method with $h = 0.2$ to estimate $y(1.4)$.

$5+20$

$y(1.2) = y_1 = y_0 + f(x_0, y_0) \cdot h$

$= -2 + (5 \cdot 1 - 10 \cdot (-2)) \cdot 0.2$

2 $= -2 + 25 \cdot 0.2 = 3$ 1

$6 - 30$ -24 -4.8

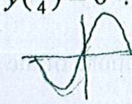
$y(1.4) = y_2 = y_1 + f(x_1, y_1) \cdot h = 3 + (5 \cdot 1.2 - 10 \cdot 3) \cdot 0.2$ *

$= 3 - 4.8 = -1.8$ 1

What does the Existence & Uniqueness Theorem tell you about the IVP $(\sin x)y' - y^{\frac{5}{2}} = 0, \quad y(\frac{\pi}{4}) = 0$? SCORE: $1\frac{1}{2}$ / 3 PTS

Justify your answer properly, but briefly.

$f = y' = \frac{y^{\frac{5}{2}}}{\sin x}$ $\frac{1}{2}$ continuous



$f_y = \frac{\frac{5}{2} y^{\frac{3}{2}}}{\sin x}$ 1

$= \frac{5\sqrt{y^3}}{2\sin x}$

interval:

$x \in (0, \pi)$

$y \in (0, +\infty)$ it has a solution and

$y(\frac{\pi}{4}) = 0$: the solution of this DE has unique

Consider the DE $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^2$. SCORE: 6 / 6 PTS

[a] Is $y = Ax^3 + x^2 + Bx$ a family of solutions of the DE ?

$\frac{dy}{dx} = 3Ax^2 + 2x + B$

$\frac{d^2y}{dx^2} = 6Ax + 2$

$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^2(6Ax+2) - 2x(3Ax^2+2x+B) + 2(Ax^3+x^2+Bx)$ 1 1 1

$= 6Ax^3 + 2x^2 - 6Ax^3 - 4x^2 - 2Bx + 2Ax^3 + 2x^2 + 2Bx$

$1\frac{1}{2}$ $= 2Ax^3 \neq x^2$

[b] If the answer to [a] is "YES", solve the IVP consisting of the DE and the initial conditions $y(1) = -1, \quad y'(1) = 3$.
If the answer to [a] is "NO", skip this part.

It's not a solution $1\frac{1}{2}$